The Bell System Technical Journal

Vol. XXII *October, 1943* *No. 3*

Effect of Feedback on Impedance

By R. B. BLACKMAN

THE impedance of a network is defined as the complex ratio of the alternating potential difference maintained across its terminals by an external source of electromotive force, to the resulting current flowing into these terminals. If the network contains active elements such as vacuum tubes, the resulting current (or potential difference if the input current is taken as the independent variable) may be due in part to the excitation of the active elements. The definition of impedance does not discriminate between the part of the current (or potential difference) due directly to the external source of electromotive force and the part due to the excitation of the active elements by the external source. Hence the impedance will in general depend upon the degree of activity of the active elements.

These observations were made early in the development of feedback amplifiers by H. S. Black[1] who made two important uses of the effect of feedback on impedance. In the first place it afforded a method of measuring feedback which has some advantages over the method which involves opening the feedback loop, providing proper terminations for it and measuring the transmission around it. In the second place the effect of feedback on impedance was used to control the impedances presented by a feedback amplifier to the external circuits connected to it.

Relations between impedance and feedback were derived by Black and others for a number of specific feedback amplifier configurations. In some cases these relations turned out to be very simple. For the most part, however, these relations were so complicated that they defied reduction to a common form.[2] The difficulty seems to have been due, in part at least, to the attempt to formulate the relationship, in each case, in terms of the normal feedback of the amplifier. In some cases the difficulty seems to have been due partly also to the valid, but, as it turns out, irrelevant observation that the feedback is affected by the impedance of the measuring circuit as

[1] H. S. Black, "Stabilized Feedback Amplifiers", *B.S.T.J.*, January, 1934.

[2] Shortly after the general relationship between feedback and impedance was derived, it was independently established by H. W. Bode and J. M. West by examination of a variety of feedback amplifier designs. The generality of the relationship was also independently proved for amplifiers with a single feedback path by J. G. Kreer and by C. H. Elmendorf.

well as by the removal of any impedance elements or circuits which are normally connected to the amplifier.

These difficulties are avoided by the method of derivation adopted in this paper. Illustrative examples are then given of some of the uses to which the general relationship between feedback and impedance may be put.

Derivation

The derivation of the general relationship between feedback and impedance will be made here with reference to the diagram shown in Fig. 1.

One of the vacuum tubes in the network, namely that one to which the feedback is to be referred, is shown explicitly at the top of the box in the diagram. The grid lead to this tube is broken at terminals 2, 2′. In practice, the hreak in the grid lead would leave the grid still coupled to some

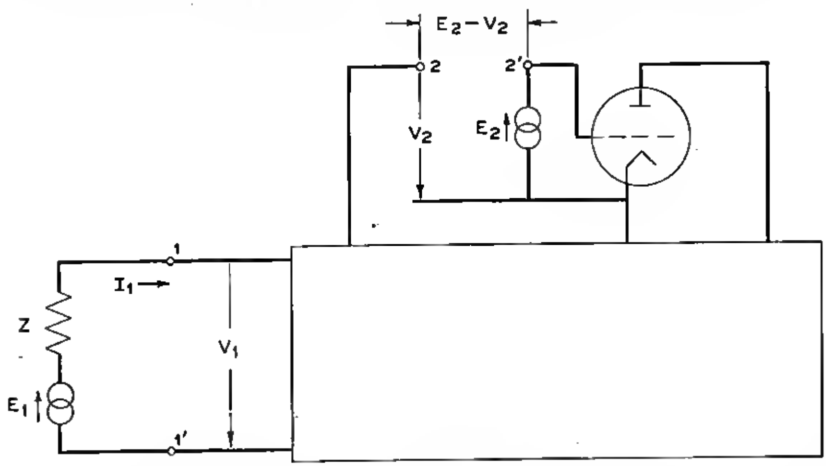

Fig. 1—Relation between feedback and impedance.

degree to the other electrodes of the tube through parasitic interelectrode admittance. For analytical purposes, however, it may be assumed that the parasitic admittances between the grid and the other electrodes of the tubes are connected not directly to the grid within the tube but to some point farther out along the grid lead. Under this assumption the break in the grid lead not only removes the feedback to the tube completely, but also leaves the parasitic admittances connected in the network in such a way that their contribution to the feedhack is implicitly taken into account. Furtbermore, the impedance looking into the grid of the tube is now infinite so that if a voltage is applied to the grid no current will be drawn from the source of the voltage.

At the left-hand side of the box in the diagram, terminals 1, 1′ are brought out. These are the terminals to which the impedance is to be referred. In

the normal condition of the network these terminals may be connected through an external impedance branch. This is the case, for example, when terminals 1, 1′ are the input terminals of a feedback amplifier whose input impedance is under investigation. However, this external impedance may also be zero or infinite according as terminals 1, 1′ are "mesh-terminals" obtained by breaking open a mesh of the network, or "junction-terminals" obtained by bringing out two junctions of the network.

It is assumed that the network, including all of the vacuum tubes, is a linear system in which, therefore, the Superposition Principle holds. Hence, if an e.m.f. E_1 is applied in series with terminals 1, 1′ and a second e.m.f. E_2 is applied between the grid and the cathode of the tube, the potential difference V_1 developed across the input terminals 1, 1′ and the potential difference V_2 developed between the terminal 2 and the cathode of the tube will be linearly related to E_1 and E_2. If the source of E_1 has internal impedance the coefficients in these relations will depend upon this impedance. However, if the input current I_1 is used as an independent variable in place of the e.m.f. E_1 the coefficients will not depend upon the impedance of the source of the current I_1. It is also convenient to consider the potential difference $E_2 - V_2$ developed across the terminals 2, 2′ as one of the dependent variables in place of V_2. Therefore,

$$\left.\begin{aligned} V_1 &= AI_1 + BE_2 \\ E_2 - V_2 &= CI_1 + DE_2 \end{aligned}\right\} \tag{1}$$

where the coefficients are independent of Z.

From these equations we obtain

$$\left(\frac{V_1}{I_1}\right)_{E_2=V_2} = \frac{AD - BC}{D}$$

$$\left(\frac{V_1}{I_1}\right)_{E_2=0} = A$$

$$\left(\frac{E_2 - V_2}{E_2}\right)_{V_1=0} = \frac{AD - BC}{A}$$

$$\left(\frac{E_2 - V_2}{E_2}\right)_{I_1=0} = D$$

Hence

$$\frac{\left(\dfrac{V_1}{I_1}\right)_{E_2=V_2}}{\left(\dfrac{V_1}{I_1}\right)_{E_2=0}} = \frac{1 - \left(\dfrac{V_2}{E_2}\right)_{V_1=0}}{1 - \left(\dfrac{V_2}{E_2}\right)_{I_1=0}} \tag{2}$$

Tbis equation expresses the relationship between feedhack and impedance. To make this more apparent tbe physical significance of each of the factors in tbis equation will be examined and suitable symbols will be substituted for tbem.

In equations (1) E_2 and I_1 were regarded as independent variables. However, the ratio $\left(\frac{V_1}{I_1}\right)_{E_2=V_2}$ implies that E_2 is adjusted to be equal to V_2. This means that E_2 is dependent upon I_1. The reason for the imposition of this dependence is that with E_2 equal to V_2 the terminals 2, 2′ may be connected together and the source of E_2 may be removed without affecting, in particular, the potential difference V_1 across terminals 1, 1′ and the current I_1 into these terminals.

Obviously, therefore, the ratio $\left(\frac{V_1}{I_1}\right)_{E_2=V_2}$ is the impedance which will be seen at the terminals 1, 1′ when terminals 2, 2′ are connected together and the only source of e.m.f. acting on the network is tbe external circuit connected to the terminals 1, 1′. This ratio will be symbolized by Z_A.

The ratio $\left(\frac{V_1}{I_1}\right)_{E_2=0}$ implies that no voltage is applied between the grid and the cathode of the tube. However, it is immaterial whether or not a voltage is applied to the grid of the tube if the amplification of the tube is nullified. Obviously, therefore, this ratio is the impedance which will be seen at the terminals 1, 1′ when terminals 2, 2′ are connected together and the amplification of the tuhe is nullified. This ratio will be symbolized by Z_P.

Finally, the ratios $\left(\frac{V_2}{E_2}\right)_{V_1=0}$ and $\left(\frac{V_2}{E_2}\right)_{I_1=0}$ are readily recognized from the definition of feedback to be the feedback to the vacuum tube with the terminals 1, 1′ connected together in the first case, and left open in the second. These ratios will be symbolized by F_{Sh} and F_{Op} respectively.

Hence, equation (2) may be written in the more significant form

$$\frac{Z_A}{Z_P} = \frac{1 - F_{Sh}}{1 - F_{Op}} \tag{3}$$

Determination of Feedback

One of the uses to which the relationship (3) may be put is in the determination of feedback by impedance measurement. However, since this relationship involves two feedbacks, only one of which may be identified with the feedback to be determined, one of these feedbacks must be known.

In the most common types of feedback amplifiers it is possible to choose

terminals 1, 1′ so that either F_{Sh} or F_{Op} is zero. If $F_{Op} = 0$ and $F_{Sh} = F_N$ where F_N is the normal feedback, then

$$F_N = 1 - \frac{Z_A}{Z_P} \tag{4}$$

On the other hand, if $F_{Sh} = 0$ and $F_{Op} = F_N$ then

$$F_N = 1 - \frac{Z_P}{Z_A} \tag{5}$$

Fig. 2 shows a feedback amplifier in whicb the μ-circuit and tbe β-network are connected in series at one end and in parallel at the other end. At terminals 1, 1′ in this figure the conditions for formula (4) are obviously fulfilled. Hence, if the impedance measurements are made at these terminals, the feedback is given by formula (4). On tbe other hand, at terminals 2, 2′ in Fig. 2 the conditions for formula (5) are obviously fulfilled. Hence, if the impedance measurements are made at these terminals, the feedback is given by formula (5).

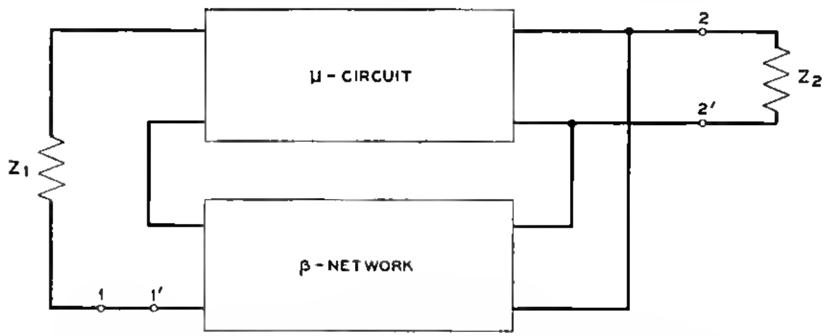

FIG. 2—Feedback amplifier with series feedback at one end and shunt feedback at the other end.

If the grid-plate parasitic admittance of a tube in a feedback amplifier is not negligible it is not possible to open any physical mesh in the amplifier so that $F_{Op} = 0$ for that tube. In such a case, therefore, (4) is not applicable. However, if the impedance measurements are made between the grid and the cathode of tbat tube the conditions for formula (5) are obviously fulfilled, and the feedback is given by formula (5). Hence, of the two particular forms (4) and (5) of the general relationship (3), only (5) enjoys complete generality in the determination of feedback by impedance measurements.

Feedback during Impedance Measurements

While the feedback computed from impedance measurements by formula (4) or (5) is the normal feedback, the feedback during the impedance measurements may be quite different, due to the impedance of the impedance measuring circuit. Referring to Fig. 1 we see that the feedback during measurement is by definition

$$F_Z = \left(\frac{V_2}{E_2}\right)_{V_1 = -ZI_1}$$

where Z is the impedance of the impedance measuring circuit. By equations (1) this is easily reduced to

$$F_Z = \frac{Z_P F_{Sh} + Z F_{Op}}{Z_P + Z} \tag{6}$$

Under the conditions to which formula (4) applies

$$F_Z = \frac{F_N}{1 + \dfrac{Z}{Z_P}} \tag{7}$$

It is clear therefore that even if F_N satisfies Nyquist's Stability Criterion, Z may be of such a character that F_Z violates that criterion. In that case it will be impossible to make the impedance measurements.

Contrariwise, if F_N violates Nyquist's Stability Criterion, it is possible to choose Z so that F_Z satisfies that criterion and make it possible to measure the impedance. Substituting (4) into (7) we find that a sufficient but not necessary condition in order that $|F_Z| < 1$ is that

$$|Z| > |Z_A| + 2|Z_P|$$

Under the conditions to which formula (5) applies

$$F_Z = \frac{F_N}{1 + \dfrac{Z_P}{Z}} \tag{8}$$

Similar observations may be made with respect to (8) as were made with respect to (7). Substituting (5) into (8) we find that a sufficient but not necessary condition in order that $|F_Z| < 1$ is that

$$\frac{1}{|Z|} > \frac{1}{|Z_A|} + \frac{2}{|Z_P|}$$

Feedback Control of Impedance

The application of the relationship (3) to the feedback control of impedance may be illustrated by a few concrete examples.

Let us assume that we are interested in the impedance faced by the line impedance Z_1 in Fig. 2. If the terminals 1, 1′ in Fig. 3 are left open the feedback is obviously zero. Let the feedback when the terminals are shorted together be denoted by F_{Sh}. If the impedances of the μ-circuit and the β-network are denoted by Z_μ and Z_β, respectively, then

$$Z_A = Z_P(1 - F_{Sh}) \tag{9}$$

where

$$Z_P = Z_\mu + Z_\beta$$

This shows the now well-known fact that series feedback may be used to magnify impedance.

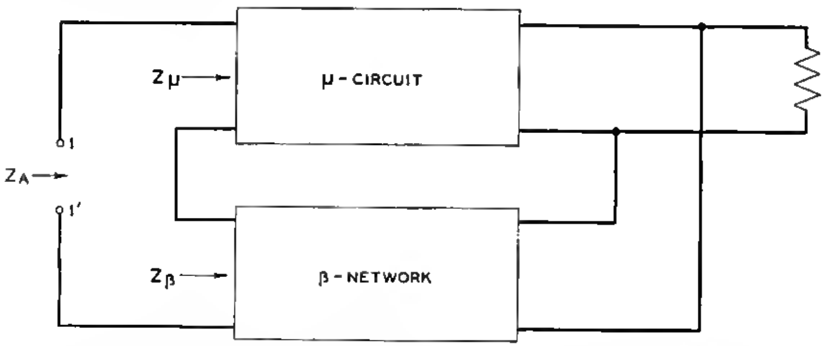

FIG. 3—Impedance faced by the line at the series feedback end of a feedback amplifier.

However, it should be noted that the feedback F_{Sh} involved in (9) is not now equal to the normal feedback F_N as it was when the terminals 1, 1′ were taken as in Fig. 2. The relation between F_N and F_{Sh} may be obtained from (6) by identifying F_N with F_Z, and Z_1 with Z. Hence

$$F_N = \frac{F_{Sh}}{1 + \dfrac{Z_1}{Z_P}} \tag{10}$$

From (9) and (10) it follows that even with a very modest amount of normal feedback the magnification of the impedance may be very large. For example, if $Z_P = 1000$ ohms, $Z_1 = 1$ megohm and $F_{Sh} \doteq -1000$, then Z_A is better than 1000 times as large as Z_P although F_N is not quite unity in magnitude.

Similarly, the impedance faced by the line impedance Z_2 in Fig. 2, as shown in Fig. 4, is

$$Z_A = \frac{Z_P}{1 - F_{Op}} \tag{11}$$

where

$$Z_P = \frac{Z_\mu Z_\beta}{Z_\mu + Z_\beta}$$

This shows the now well-known fact that shunt feedback may be used to reduce impedance.

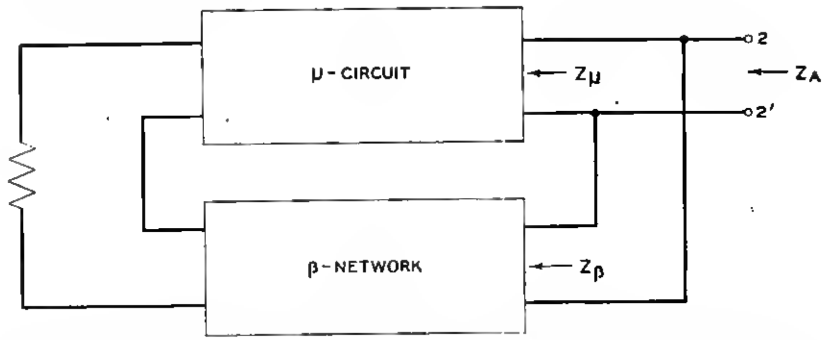

FIG. 4—Impedance faced by the line at the shunt feedback end of a feedback amplifier.

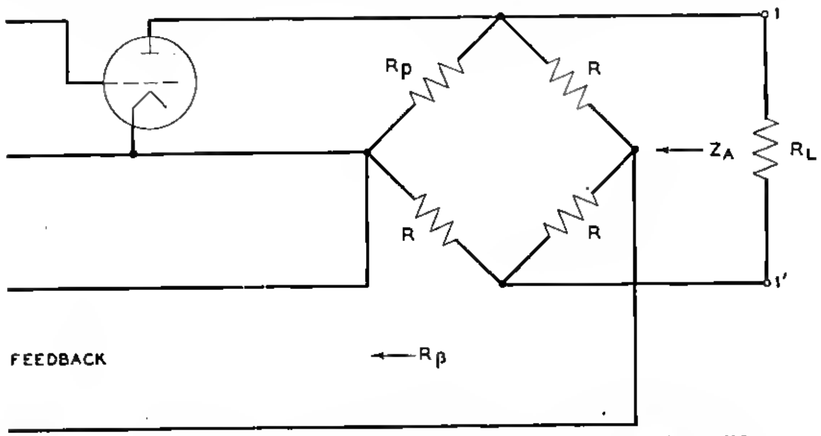

FIG. 5—Impedance faced by the line connected to a bridge feedback amplifier.

The relation between the normal feedback F_N and the feedback F_{Op} involved in (11) is, by (6)

$$F_N = \frac{F_{Op}}{1 + \frac{Z_P}{Z_2}} \tag{12}$$

From (11) and (12) it follows that even with a very modest amount of normal feedback the reduction in impedance may be very large. For example,

if $Z_P = 100{,}000$ ohms, $Z_2 = 100$ ohms and $F_{O_p} = -1000$, then Z_A is less than 100 ohms although F_N is not quite unity in magnitude.

The two examples given above illustrate the use of feedback to magnify or to reduce the impedance of a network. This impedance, however, will be correspondingly sensitive to changes in the characteristics of the vacuum tubes. A third example of the use of the relationship (3) will show that feedback may also be used to make the impedance of a network less sensitive to changes in the characteristics of the vacuum tubes.

In the case of the bridge-type feedback network shown in Fig. 5 we have, with respect to the terminals 1, 1′

$$Z_P = R(1 + Q)$$

$$F_{Sh} = A\,\frac{1 + \dfrac{2R + 3R_\beta}{R + R_\beta}Q}{1 + Q}$$

$$F_{O_p} = A\left(1 + \frac{2R + 3R_\beta}{R + R_\beta}Q\right) = (1 + Q)F_{Sh}$$

where A is the feedback designed for the condition $R_p = R$, and

$$Q = \frac{(R_p - R)(R + R_\beta)}{(R + R_p)(2R + R_\beta) + 2RR_\beta}$$

Then, by (3)

$$Z_A = R\left(1 + \frac{Q}{1 - F_{O_p}}\right)$$

Hence, if the feedback F_{O_p} is very large the effect of bridge unbalance on the impedance presented to the line will be very small. If, for example, the design feedback is 40 db the output impedance cannot change more than 1 per cent however severely the bridge might be unbalanced by R_p being larger than R.

The feedback when the line impedance R_L is connected may be obtained by identifying R_L with Z in formula (6). It is

$$F = A\,\frac{1 + \dfrac{2R + 3R_\beta}{R + R_\beta}Q}{1 + \dfrac{R}{R + R_L}Q}$$

whence

$$\frac{\partial \log F}{\partial \log R_L} = \frac{RR_L}{R + R_L}\,\frac{Q}{R_L + Z_P}$$

The effect of bridge unbalance is to make the feedback sensitive to changes in the line impedance R_L.